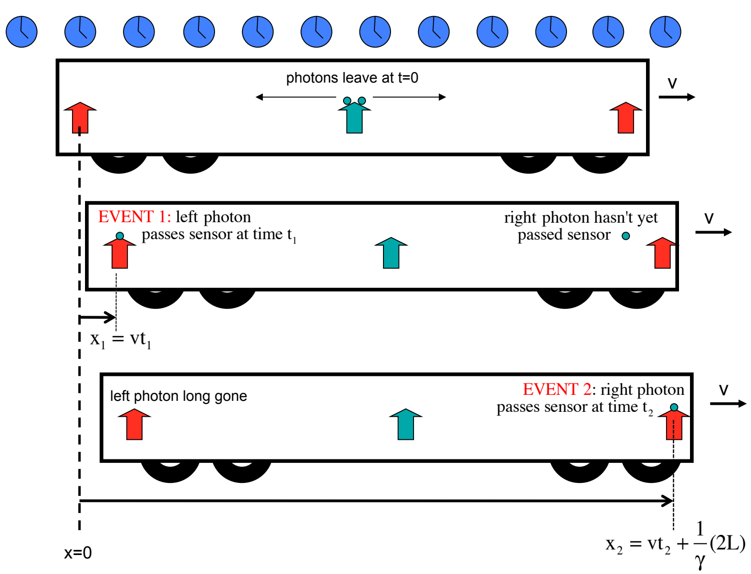
HOW DO WE DEDUCE THE GRID FOR A SPACE-TIME DIAGRAM?

The QUICK version: What you are looking at below is a graphic summary of the information you are about to be given. As you read, refer back to these sketches.



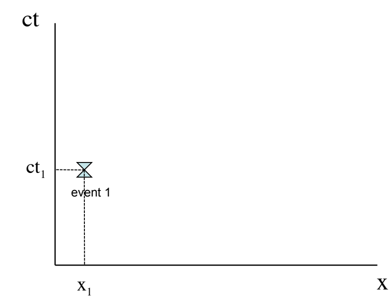
The set-up: Two photons are fired from the center of a train car moving with velocity *v*, one toward a sensor at the left end and one toward a sensor at the right end.

You are standing still by the track. Your clock starts when the photons are fired. Your unprimed, stationary coordinate origin is defined as x = 0 where the left sensor was when the photons were fired.

A certain amount of time t_1 passes by during which the left photon reaches the left sensor. This is event 1.

The train and left sensor moves a distance $x_1 = vt_1$ during that time, so from the perspective of the unprimed ("stationary") frame of reference, the unprimed coordinates for the first event are $t = t_1$ and $x = x_1 = vt_1$.

On our unprimed grid, this can be graphed as shown to the right.



During time t_1, the right-hand photon has still not reached its sensor.

After time t_2, the right photon reaches its sensor. This is event 2.

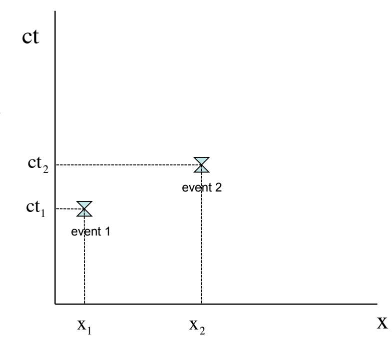
During that time, the train has traveled a distance $x_2 = vt_2$. The coordinate of the second sensor at that time will be $x_2 = vt_2$ plus the length contracted length of the car (it is located at the opposite end of the car from the left sensor). That coordinate is:

$$x_2 = vt_2 + \left(\frac{1}{\gamma}\right)(2L)$$

The event 2 coordinates are, therefore:

$$t = t_2 \text{ and } x = x_2 = vt_2 + \left(\frac{1}{\gamma}\right)(2L).$$

On our unprimed grid, this additional information is shown on the graph to the right.

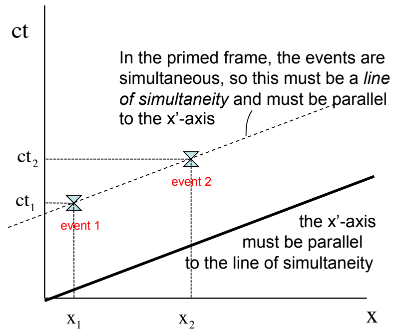


So here's the fun.

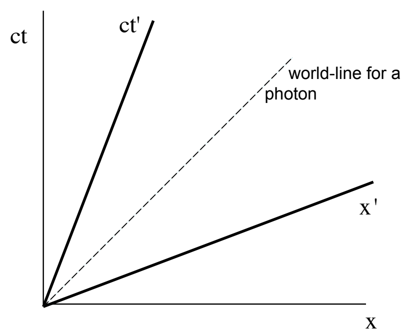
We can draw a ling between event 1 and event 2.

If we do this, we generate an interval that will not change no matter what frame of reference the events are viewed from. What is cool, and this is the crux of the matter, is that the two events happen simultaneously in the car's frame of reference (if you are in that frame, the photons leave at the same time, they travel the same distance and they pass the sensors at the same time . . .). That means the line connecting the two events must be on a line of simultaneity in that frame, which means that line must be *parallel to the x' axis*.

Drawing in that parallel line through the origin allows us to create the primed coordinate spatial axis.



What's more, because the axes must be symmetric around the forty-five degree world-line of a photon, we can generate a complete *position* and *time* grid for the primed coordinate system. That is shown below.



WHAT FOLLOWS IS A MORE EXPANDED EXPLANATION OF ALL OF THIS.

CONSTANT VELOCITY MOTION

You are a passenger in a car. You have just awakened. It is night, so you really can't see the landscape outside the car. As you sit there, half asleep and half awake, a thought pops into your head.

What a marvel! Even though I am sitting in this speeding vehicle, I don't feel any different than I did when the car was stopped.

You pull out a soft drink and pour some into a cup. To hit the cup, you don't have to compensate for the fact that the car is moving--you don't have to hold the cup back just a bit for the fluid to fall appropriately. It follows the same parabolic arc it takes when the car is stationary.

The point? Assuming you aren't accelerating, there is no experiment you can do in a confined space that will tell you if you are moving with a constant velocity or not. You *can* tell if your velocity is *changing*. An increase in speed will throw your head backward and a slowing will throw your head forward. But if you are blindfolded and moving with a constant velocity, you have no way of telling whether you are going 60 mph (car travel), 600 mph (jet travel) or zero mph (sitting like a slug on the tarmac).

FROM WHENCE COMES THE PRIME COORDINATE GRID?

The plan in a nutshell:

1.) Synchronize a set of clocks in a "moving" frame of reference. Synchronize a set of clocks in a "stationary" frame. Then instigate two events that are simultaneous in the "moving" frame taking additional data for the events as viewed from the "stationary frame.

2.) Graph the "stationary" frame's data.

3.) The interval between the two events is fixed. Because the two events happened simultaneously in the "moving" frame, a line between those events must be parallel to its x-axis. Use this fact to draw the x-axis for the "moving" frame.

4.) Note that coordinate axes are always drawn symmetrically around the world line of a photon of light, ,and know what the spatial position axis looks like, you can now draw the time axis.

FROM WHENCE COMES THE PRIME COORDINATE GRID?

A freight train car is moving with a constant velocity “v.” You are inside the car and can’t see out. You have at your disposal colored light bulbs that can be flashed, light sensors, meter sticks and cesium (atomic) clocks.

You are primed and ready to so some serious scientific experimentation.

Your frame of reference is the train. As such, and even though it is moving relative to the outside terrain, all you are interested in is the train’s inside compartment. As such, as far as you are concerned, you, the train and the train’s interior are all stationary. We will call this the primed frame of reference.

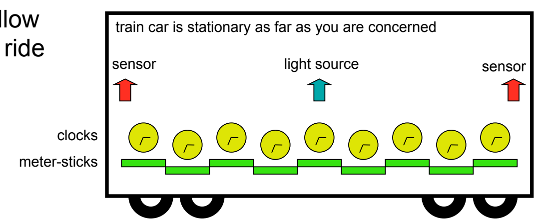
Inside the train’s “stationary” compartment, you begin by setting up the experiment.

1.) Place an aqua-colored light emitter at the center of the car (the light is turned off).

2.) Place red light-sensors at the front and rear of the car a distance “L” meters from its center.

3.) Lay down a lattice of shockingly green meter sticks.

4.) Lastly, create a lattice of garish yellow clocks that will ride along stationary relative to the car.



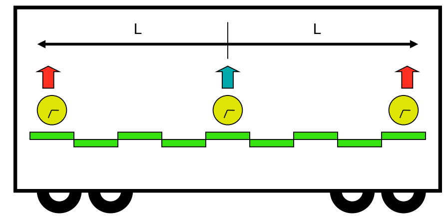
To synchronize the clocks:

1.) Fire photons simultaneously to the left and right from the central light source. The clock attached to that coordinate will register the time of firing.

2.) When each photon gets to its respective sensor, the clock at that location will register and record the time it arrives.

3.) Each sensor is a distance “L” units from the light source, so the travel time for both photon should be L/c. That means both clocks should read times that are the same, and each should have registered times that are L/c seconds later than the time registered by the central clock.

4.) If the clocks don’t so register, set them so they do.

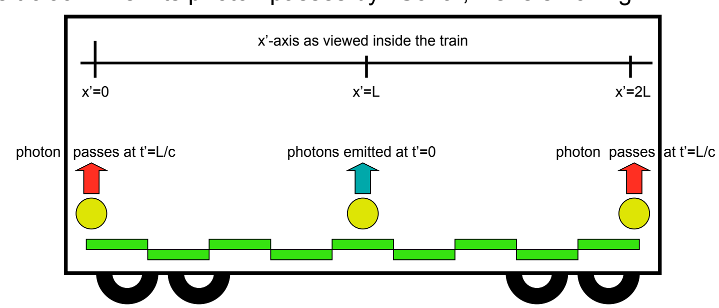


With the clocks synchronized, we can begin.

1.) Assume the train’s frame of reference is designated the “prime” axis.

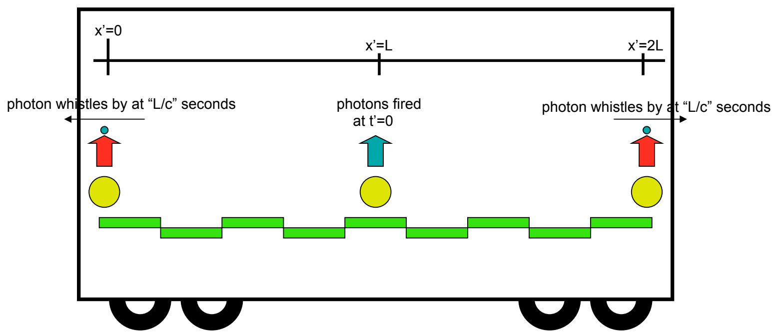
2.) Assume the t’ = 0 point is when the photons are fired from the central light source. As it takes “L/c seconds” for the photons to make it to the sensors at both ends of the car, the two clocks will register the same time as the photons pass. In other words, the events will be simultaneous.

3.) Assume the x’ = 0 (i.e., the coordinate origin) is at the position of the left clock when its photon passes by. So far, we’re smoking!



The data you generate is:

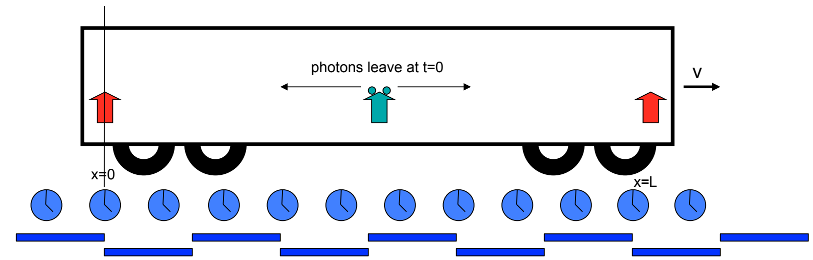
event	position	time
photon passes left sensor	x'=0	t'=L/c
photon passes right sensor	x'=2L	t'=L/c



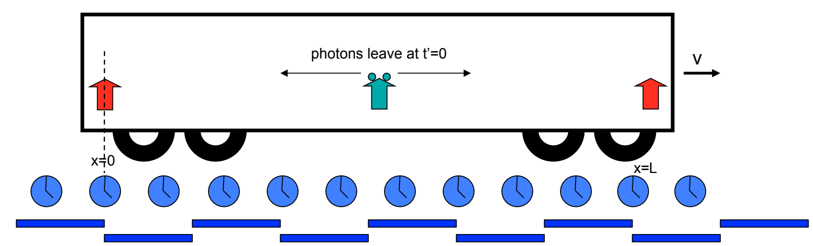
NOW COMES THE FUN ! ! !

How do things look from the perspective of an observer sitting stationary on the ground as the train passes by?

The first thing we have to do is set up synchronized clocks and set up a lattice of meter sticks attached to the ground in the "outside" frame (we will call this the "unprimed" frame of reference).



Assume the t = 0 point for this frame of reference is when the photons are first emitted from train's center, and the x = 0 point is where the train's left sensor is positioned when the photons are first emitted.



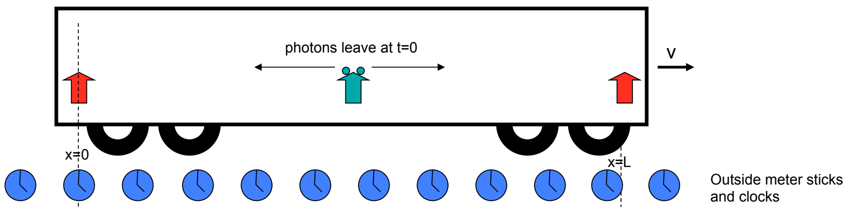
With this, the chain of events as viewed from outside the car are as follows:

a.) At t=0, the photons are emitted.

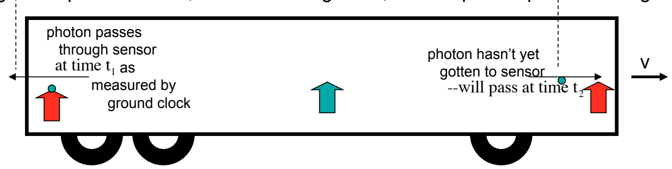
b.) While the photons move out to the right and left on their way to the red sensors, the car moves a distance d=vt (the train *is* moving with velocity "v").

c.) The outside clocks do not register the same passage times as the train is moving (see sketch). The left clock registers t_1; the right clock registers t_2.

The car's position, relative to the ground, when the photons are emitted.

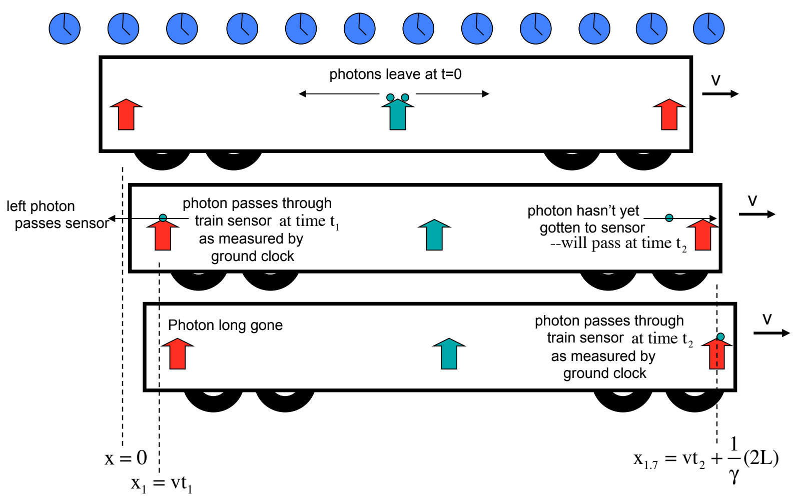


Exaggerated position of car, relative to the ground, when a photon passed through left sensor.

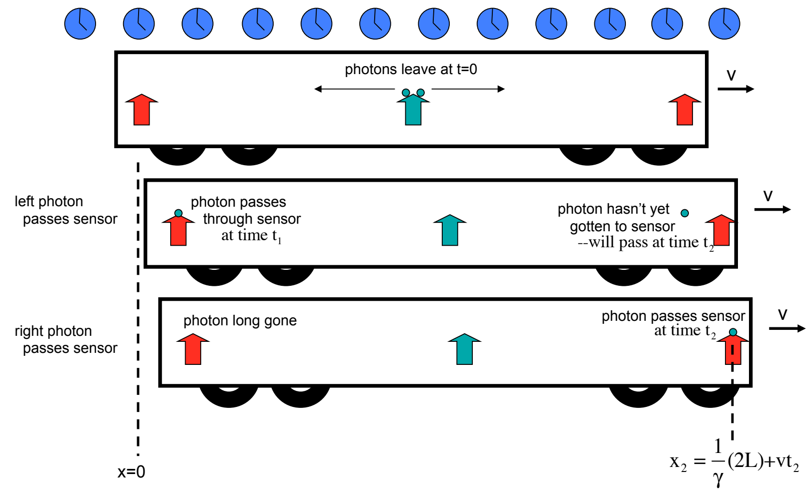


d.) When the left photon passed its sensor, $x_1 = vt_1$, where t_1 is the ground time-of-transit from the emit point. Due to length contraction, the right sensor's coordinate will be:

$$x_{1.7} = \frac{1}{\gamma}(2L)+v_1 t$$



e.) Between the time the left photon passes through its sensor and the right photon passes through its sensor, the train will have travel an additional distance and the "photon passage" coordinate will be $x_3 = \frac{1}{\gamma}(2L)+vt_2$.



So how does the data look as we compare the two situation?

As taken from the clocks and meter-sticks in the primed coordinate system (i.e., the frame fixed inside the car)

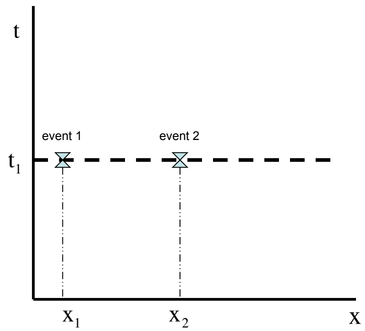
event	position	time
photon passes left sensor	x'=0	t'=L/c
photon passes right sensor	x'=2L	t'=L/c

As taken from the clocks and meter-sticks in the unprimed coordinate system (i.e., the frame fixed to the ground outside the car)

event	position-- this happens at:	time
photon passes left sensor	$x_1 = vt_1$	t_1
photon passes right sensor	$x_2 = \frac{1}{\gamma}(2L) + vt_2$	t_2

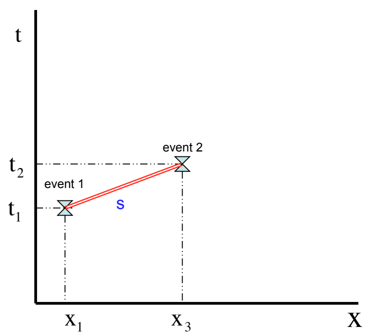
Minor Side Point: If two events happen at the same time at two points in space, and if you show those events on a graph (see below), a line between the two events will define what is called a “line of simultaneity. That line will ALWAYS be parallel to the x-axis.

Bottom line: Lines of time simultaneity are ALWAYS PARALLEL to the x-axis. Remember this: No matter what a coordinate axis looks like, lines of simultaneity will ALWAYS be parallel to the x-axis.



Let’s graph the data associated with the ground frame:

event	position	time
photon passes left sensor	$x_1 = vt_1$	t_1
photon passes right sensor	$x_2 = \frac{1}{\gamma}(2L) + vt_2$	t_2

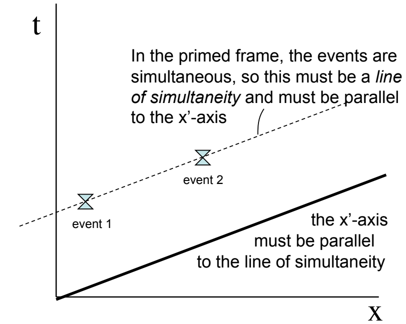


In a nutshell, event 1 occurred at t_1 when the sensor’s coordinate was $x_1 = vt_1$, and event 2 occurred at t_2 when the sensor was at coordinate $x_2 = \frac{1}{\gamma}(2L) + vt_2$.

The interval s between those two events is shown by a double line in the sketch. The interval is a fixed quantity and does not change from frame of reference to frame of reference.

So this is what’s cool. In the “moving,” train frame (the primed frame), the two events we graphed happen SIMULTANEOUSLY! That means that if we draw a line between the events, that line must be parallel to the x’-axis. In other words, by identifying the events and interval using data from the unprimed frame of reference, we can cleverly determine the orientation of the primed spatial x’-axis as it exists relative to the unprimed axes (see graph below).

event	position	time
photon passes left sensor	x’=0	t’=L/c
photon passes right sensor	x’=2L	t’=L/c

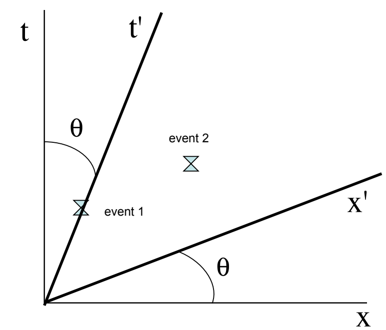


To deduce the time axis simply:

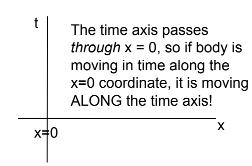
a.) From symmetry, the time axis will proceed as a mirror image of the position axis.

To deduce the time axis more rigorously:

a.) Relative to the ground (i.e., relative to the unprimed frame of reference), the left sensor is traveling at velocity “v.” If we graph that sensor’s world line, it will proceed at some angle θ relative to the unprimed time axis.



b.) But every point on that line will have a in-train (primed) coordinate of x’=0, so that line will be the position equivalent to a line of simultaneity (this is sometimes known as a *line of common position*).

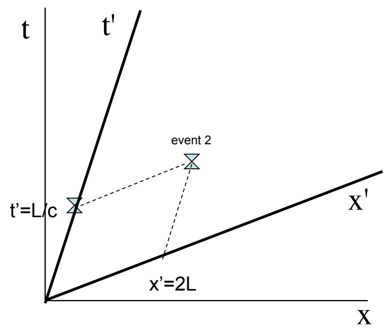


c.) Being such, that line will be parallel to the time axis.

d.) But, on graphs the time axis passes *through* x = 0, so that line must also represent the time axis. QED

So let's say we are given the graph shown below. The x-coordinate for event 2 is found by following a line parallel to the time axis back to the x-axis and reading off the number on that axis. As similar approach is used to determine the time of event 2. The graph shows both of these processes.

event	position	time
photon passes left sensor	x'=0	t'=L/c
photon passes right sensor	x'=2L	t'=L/c

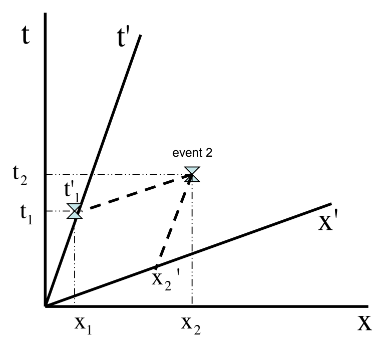


NOTE: To check, look at the graph and read off the coordinates for both events. You should find they match the values in the table.

Putting it all together, we see both axis presented as shown:

event	position	time
photon passes left sensor	x'=0	t'=L/c
photon passes right sensor	x'=2L	t'=L/c

event	position	time
photon passes left sensor	$x_1 = vt_1$	t_1
photon passes right sensor	$x_2 = \frac{1}{\gamma}(2L) + v\Delta t$	t_2



Again, check by looking at the graphs and reading off the coordinates for both events as presented in each frame. You should find the match the values in the table.

Minor note: If you want a time measured in *meters of light time* (units *meters*) but are dealing with time in *seconds*, the way one converts from *seconds* to *meters of light time* is to multiply time by the speed of light *c*. That is:

$$(c\ \text{meters}/\cancel{\text{second}})(t\ \cancel{\text{seconds}}) = ct(\text{meters})$$

As most actual time data is presented in *seconds*, the appropriate unit presentation for time on a space-time graph is "ct" and a standard presentation of the ideas outlined on the previous pages should look like the graph to the right.